USA Energy Dependent Instrumental Effect: The Electronics Pile-Up Problem

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1. Introduction

This document will discuss the know aspects of the bump and dip found in the USA power spectra. Figure 1 shows a typical dip in the data. Figure 2 shows a typical bump in the data.

The bump/dip has been seen in Cyg X-1, Crab, on ground iron source data and more. These features are seen in the data for modes 1, 2 and 3. It is supposed that they would be also seen in mode 4. Mode 4 data has not been carefully looked at yet.

Currently, Jeff and Elliott are working on a simulation. Until then we will have to settle for the recipe laid out in section 3. If anyone can remind me of other things that were done to test EDIE's characteristics please let me know and I will include them here.

Berrie has recently been able show how pile up is affecting the mean value of USA energy spectra. For me his findings strongly suggest that EDIE is caused by pile-up. This will be discussed in section 2

In this document:

Section 2 will show some of what Berrie has done.

Section 3 gives a recipe for correcting for EDIE.

Section 4 shows how I used XTE J1859 data to convince myself that the recipe in section 3 is legitimate.

2. Is EDIE Due to Pile-Up?

Kent was concerned that we didn't know for sure that EDIE was due to Pile-Up. So he made the suggestion that two energy spectra be created. These two energy spectra would

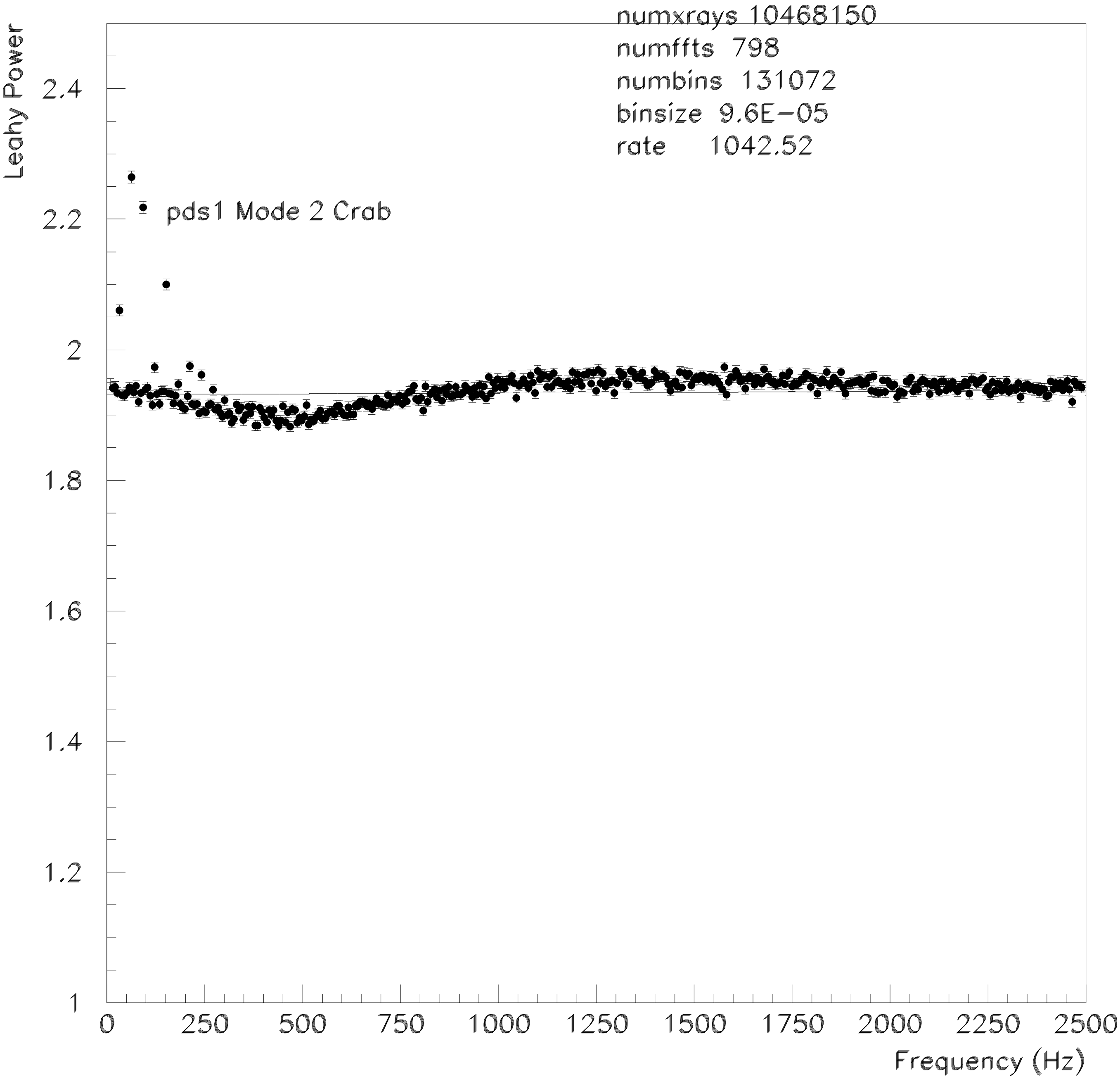

Fig. 1.— Here is an example of the Dip.

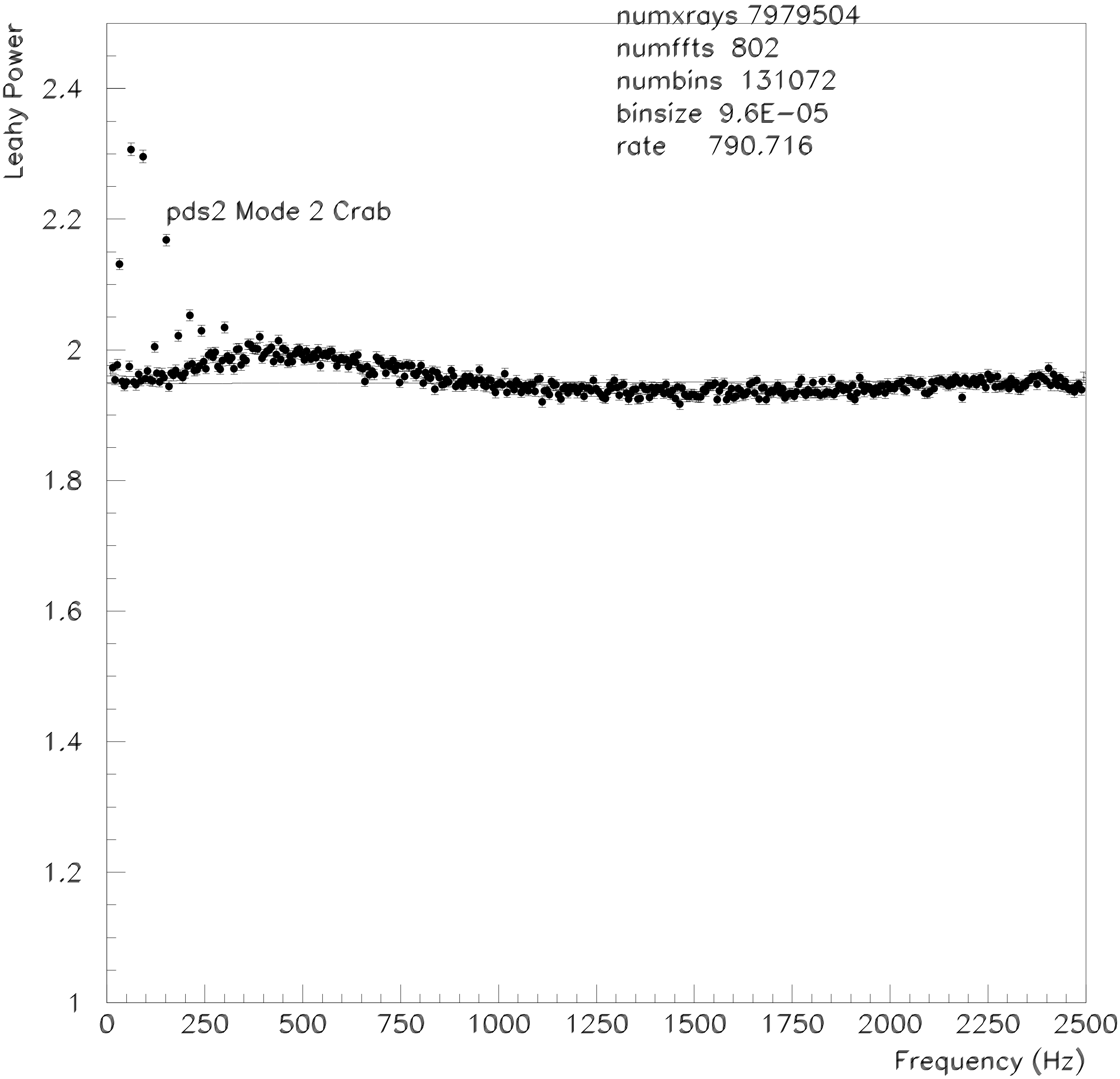

Fig. 2.— Here is an example of the Bump.

be made according to a photon to be used and its temporal proximity to the preceding photon. We tried this for several different time scales but within uncertainties we were not able to show that there was a definitive and predictable shift in the energy spectra. We definitely saw an effect; but, it was not conclusive.

Berrie then created figure 3. This figure was created by choosing all photons that were preceded by another photon in less than $96\mu\ s$ making an energy spectrum of those photons, then all photons that were preceded by another photon in less than $2 \times 96\mu\ s$ and greater than $96\mu s$ making an energy spectrum of those, etc. In the end there is and energy spectrum for all multiples of of the $96\mu s$ separation distance between adjacent photons. Figure 3 plots the average pha channel of the histogram. The average for each tick on the x-axis is calculated in the following way. For the first tick, only the average of the histogram for photons $1 \times 96\mu s$ after their preceding photon is used in the average, for the second tick the $1 \times 96\mu s$ and $2 \times 96\mu s$ histograms are used for the average, for the third tick the $1 \times 96\mu s$ and $2 \times 96\mu s$ and $3 \times 96\mu s$ histograms are used. These average values are plotted on the y-axis.

Note, figure 3 matches almost identically the shape of the autcorrelation function in Ganya's thesis. In any case, this figure gives the approximate *average* shape and magnitude of the electronics baseline. Average because the shape in figure 3 does not account for the fact that the preceding photon was not necessarily the only photon tweaking the base line. There could have been several photons in close proximity all of which affected the baseline.

If one wishes to avoid seeing an average shape of how the baseline is changed, then the same process must be repeated.(That is, if one wants to see the effect on the baseline due to only one photon a slightly different technique must be used.) This time the only photons used in an energy spectra should be those where the preceding photon is the correct distance away and the photon preceding the preceding photon came more than

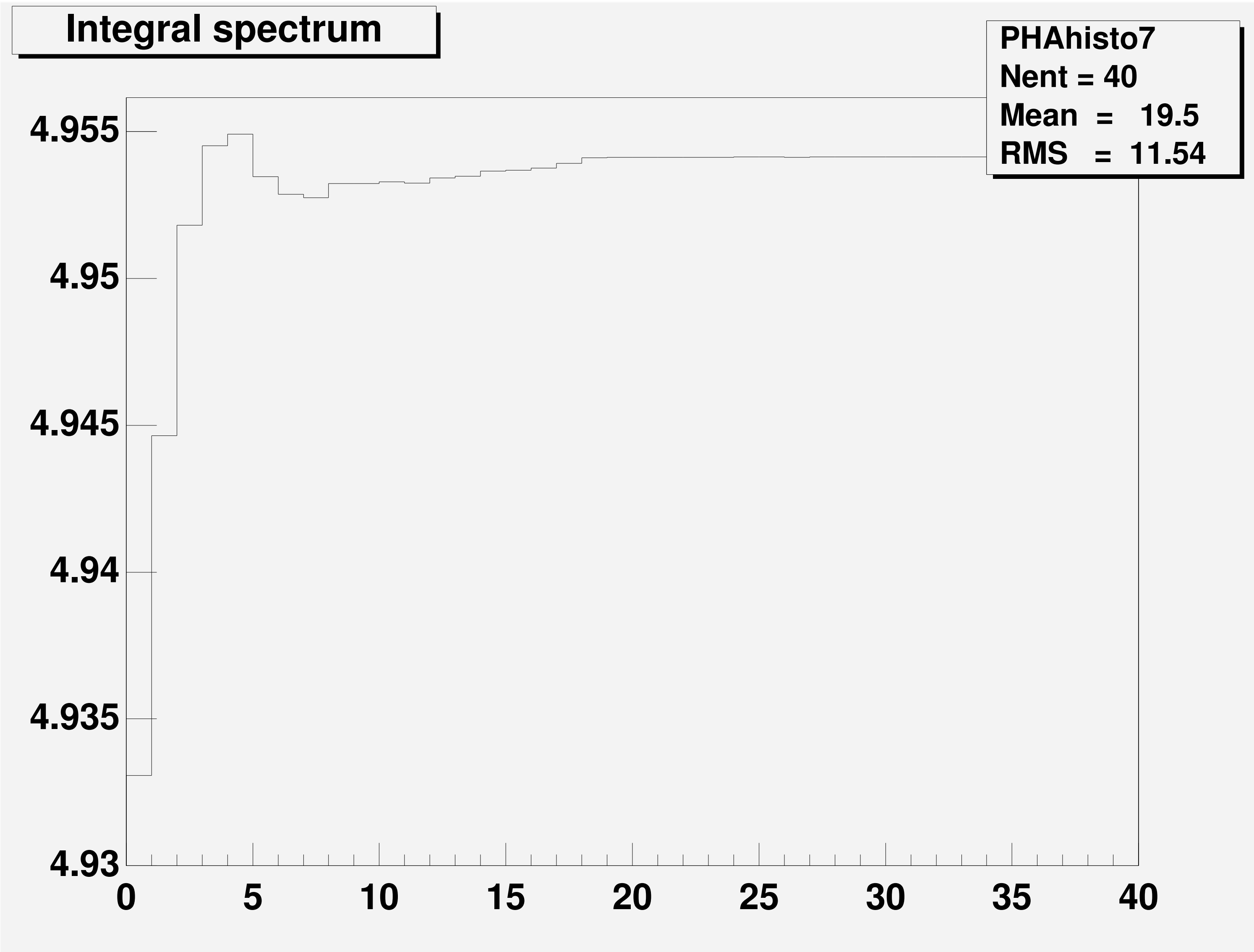

Fig. 3.— Berrie Fig. The y-axis is the average value of the energy spectrum created by photons which followed their preceding photon by a multiple of $96\mu\ s$. The x-axis is the multiple of $96\mu\ s$.

$400\mu\ s$ before the preceding photon in question. By doing this, the baseline bounce should be due to only that one photon and not earlier photons. This is exactly the plot shown in figure 4 and figure 5. The method to get figure 4 and figure 5 is the same as that for figure 3 except that multiples of $32\mu\ s$ were used instead of $96\mu\ s$. The first bin on this plot is an average value of all of the preceding photons. It should be the exact value of the the iron peak(before considering USA gain effects). This value is our zero.

This demonstration of pile-up(ie a photon sitting on the signal of previous photons and consequently having its energy and possibly timing properties altered) is not necessarily complete evidence of the cause of EDIE. It does present a strong case.

If energy cuts are made on a source, an argument can be made using Berrie's plots which can give the results which we call EDIE. The general basis of the argument is that the two boundaries in the energy spectrum where the cuts are made have differing amounts of counts,slopes,rates??(ie it's not a flat spectrum and the cuts were choosen in places were the properties about the boundaries were different) At each boundary photons will move into and out of the the selected region in accordance with the properties of that boundary. Because, the boundaries are different there will be a net periodic flux of photons into the selected energy region. The period of this flux should be set by the pile-up effects. This could be an explanation of EDIE.

Open for discussion:

In the correction of USA power spectra: Is it better to simulate it or to do it on an observation by observation basis. Simulating it is problematic because the simulation would have to be run for all rates, wait time distributions and energy spectra of interest.

It could be easier to develop a tool that takes every observation and uses its empirical qualities to generate a power spectral correction. This might be done as ffts are generated

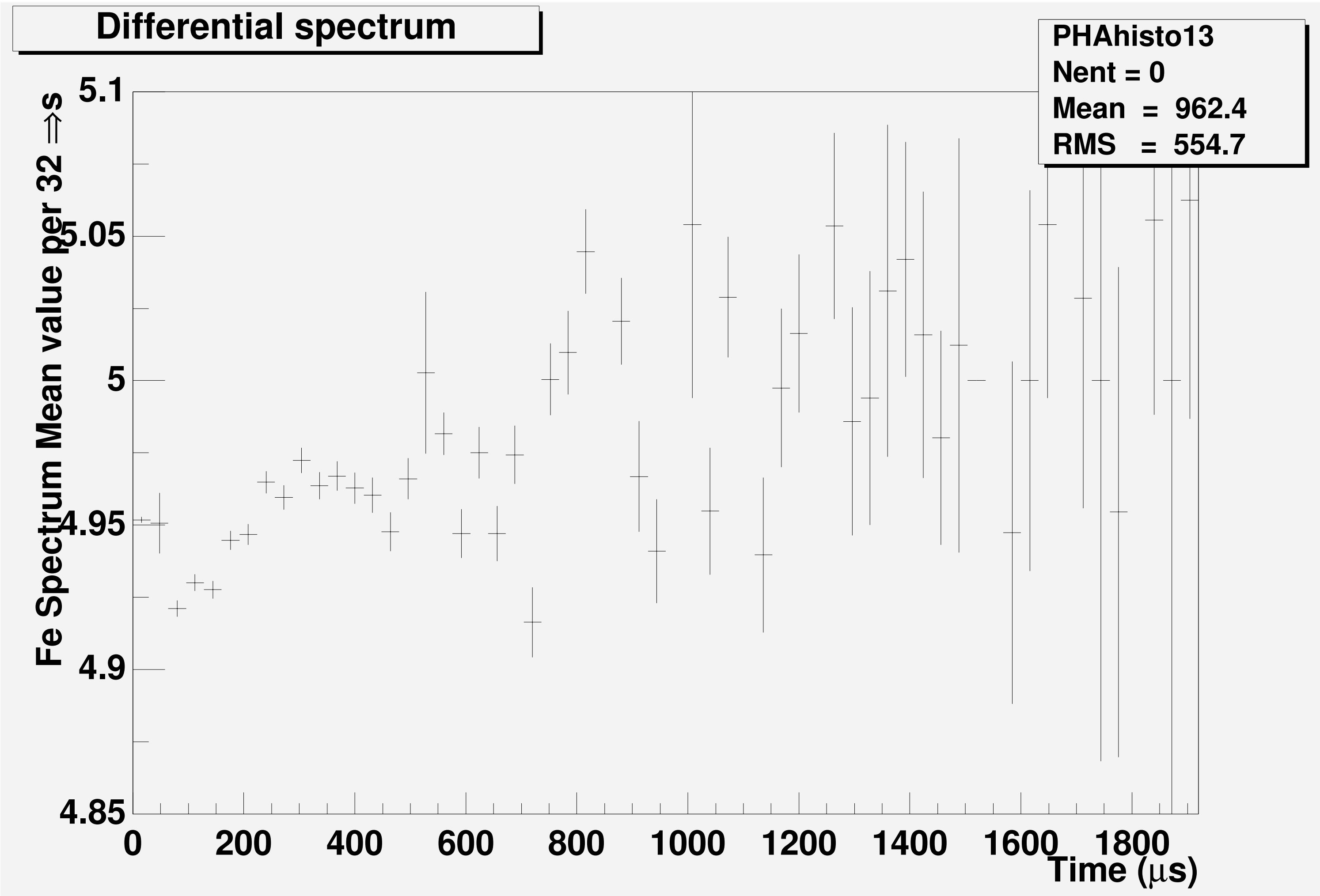

Fig. 4.— Berrie Fig.

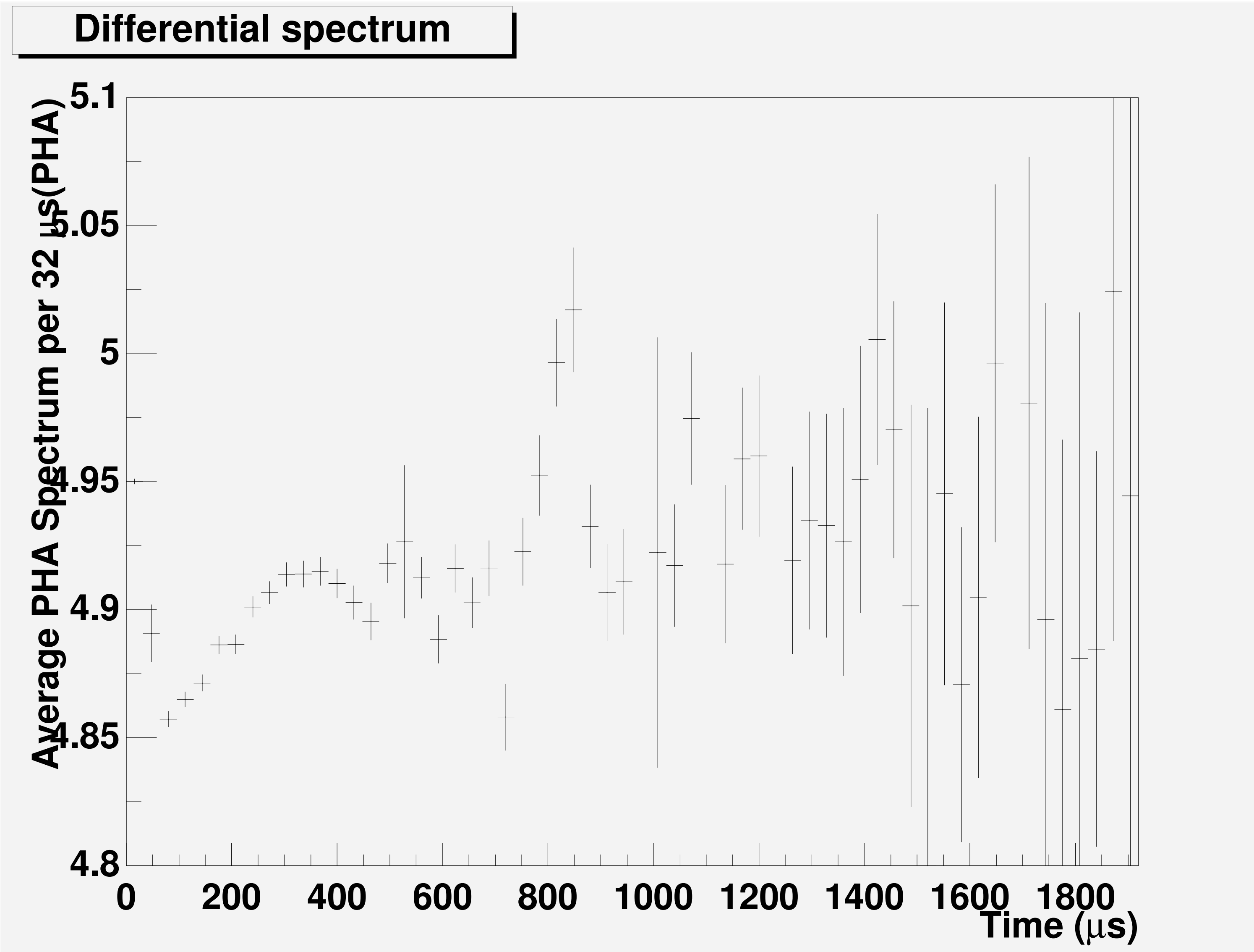

Fig. 5.— Berrie Fig.

for that observation. I am wondering if we can develop a correction technique that goes something like: 1. Create a plot like the ones in this section. 2. fit it to an appropriate polynomial(not including the first bin) 3. Scale it and give it a sign according the the energy spectral properties.

Question: how does this demonstration of pile-up effect the now existing recipe for correcting power spectra? Does it help or hurt?

3. Accounting for USA Energy Dependent Instrumental Effect

This section is taken almost verbatim from the XTE J1859 paper Appendix A.

The known qualities of the energy dependent instrumental effect, EDIE, have been learned through study of Fe^{55} ground calibration data, Cassiopeia A, the Crab Pulsar and Cygnus X-1. The general behavior of EDIE is determined by the energy spectral character and photon rate of a given source. For a more complete discussion of these dependencies please see (Shabad, G. 2000, Ph.D. Thesis, Stanford University).

The study of EDIE has made it possible to develop a preliminary but general recipe for its correction in USA power spectra. The effect is qualitatively well defined. A brief discussion of how the recipe was obtained and what the recipe is will now be given. The results of XTE J1859 data will be used as an example.

EDIE was first discovered after the USA experiment had already been launched. It is only manifest when a power spectrum is made of a source having an asymmetric energy spectrum. Note, the real energy spectrum of a source is not the issue. The issue is how the energy spectrum appears in the USA detector prior to making corrections for the USA energy response.(Thus, unless it is stated otherwise the use of the term “energy spectrum” will always refer to the uncorrected USA energy spectrum: counts seen by the detecto binned by energy channel.) Further, an asymmetric energy spectrum may be created simply by selecting on certain energy channels prior to creating a power spectrum. In fact this is how the problem first showed itself. Prior to the launch of USA, all power spectra were made using all energy channels. The primary calibration source was Fe^{55}, which has a very symmetric energy spectrum. This is why EDIE was not seen on the ground.

After the USA experiment was launched power spectra were made in which certain energy channels were cut out. The first example was seen in Cygnus X-1 when a power spectrum was made cutting out low energy channels. A very large feature was seen in the power spectrum at approximately $400Hz$. Eventually, it was found that EDIE was also present in USA Fe^{55} ground calibration data. This finding demonstrated that EDIE was an electronics problem.

Because the Fe^{55} ground calibration was taken under controlled circumstances, it was the logical starting point for characterizing the nature of EDIE. For comparison, Cassiopeia A, Crab, Cygnus X-1 and XTE J1859 data were also used as means of searching for the effect. In total, hundreds of power spectra were made. One power spectra was made for each of the 16 USA energy channels in each of the aforementioned mentioned sources. Power spectra were made for both mode 1 and mode 2 data. All power spectra depicted a qualitatively consistent story of the nature of EDIE. The following equation is a mathematical representation of that story, as developed by Gary Godfrey:

$$Power = \alpha \times EnergyPrev \times \frac{drate}{dEnergyCurr} \times rate \times StandardWiggle \tag{1}$$

where, α is an arbitrary multiplicative factor with no sign, $EnergyPrev$ is a factor which would account for the energy of the photon immediately preceding the photon of current interest(could be an average over photons??), the fraction $\frac{drate}{dEnergyCurr}$ is a representation of the slope of the energy spectrum, $rate$ is simply the rate of photons *included* in the power spectrum and $StandardWiggle$ is the would be equation describing the general shape of

EDIE in all so far observed power spectra. *Need to come back to fill in Gary's logic of this equation. Hoping Gary will write something up.*

Unfortunately, it was not possible to stick with this intuitive approach to the characterization of EDIE. When the above eqaution was fit to the data it was very difficult(so far impossible) to determine the form of $StandardWiggle$. A phenomenalogical method was then used to find a general equation for which EDIE could be described in all USA data. This equation and process, which so far has not failed, is described below.

Using Fe^{55} ground calibration data the general functional form and qualitative energy-dependence of the effect was determined. Several models were tried before settling on the following equation,

$$P^{noise}(f) = P^{instrumental}(f) + bP^{other}(f) \tag{2}$$

where

$$P^{instrumental}(f) = c_0 e^{-0.0048f'} f'(f' - c_1)(f' - c_2), \tag{3}$$

$$P^{other}(f) = P_1 + P_2 \cos(2\pi f t_b) \tag{4}$$

and

$$f' = c_3 f \tag{5}$$

P^{other} is the Poisson noise floor as modeled in **?**, f is the frequency, t_b is the fft binsize, P_1 and P_2 are parameters generated by a monte carlo simulation of the USA deadtime and its idiosyncracies. $P^{instrumental}(f)$ is the phenomenological equation obtained from the Fe^{55} data, $c_{0,1,2,3}$ and b are free parameters used in fitting an unkown power spectrum. The paramter b is approximately one in all cases so far studied. The value of -0.0048 in the exponential was found through fitting the Fe_{55} data and is thought to be a time constant of the electronics causing EDIE.

The final outcome of fitting equation 6 to the ground data was an excellent fit in the region from $30 - 5000Hz$, please see figure 10. The fit only varies from the power spectrum in the region below $30Hz$. This is completely adequate when looking for signals at higher frequencies. However, it should be noted that, for Cassiopeia A, Cygnux X-1 and XTE J1859(high state), the expected results are obtained even from $0 - 30Hz$, see **?** and figures 10 and 7, when using equation 6 to subtract the Poisson noise and instrumental effects.

Once equation 7 was obtained and modeled with several sources it then became possible to make a recipe for subtracting the USA noise floor. The procedure is to first subtract equation 8 from the spectrum, fixing b at one, and then to fit equation 7 to the power spectrum in the range from $300 - 5000Hz$.[2]. Doing so determines the parameters of equation 7 in the range of $0/30 - 5000Hz$, with possible over-subtraction of power in the range of $0 - 30Hz$. As is shown with Cassiopeia A, Cygnus X-1 and the Fe^{55} ground data, this will give an accurate description of the noise at all frequencies. Now all the parameters, $c_{0,1,2,3}$, of equation 7 are held fixed and a fit, including any structure, is made over the range $0 - 300Hz$. The fit is made to structure(qpo's, power law, etc.) plus equation 7. Finally, a fit is made to the entire power spectrum $0 - 5000Hz$ with all paramters, including structure, varying. This is done in order to obtain the most accurate χ^2 and parameter uncertainties. Using $c_{0,1,2,3}$ from this fit equation 7 is subtracted from the power spectrum.

For XTE J1859+226 in its VHS, the power spectrum was handled in the manner described above. Specifically, the power spectrum was fit with equation 7 in the range from $300 - 5000Hz$. This was done after subtracting off the dead time corrected Poisson noise

[2]The range of the fit depends on the ranges of interest. It appears that it is possible to get a very accurate model of the noise floor by fitting at higher frequencies, for example $500 - 5000Hz$

floor,$P^{Poisson}$, with $b = 1$. The resulting spectrum was then fit, in the range $0 - 300Hz$, with a power law and three Lorentzians added to equation 7; but, holding $c_{0,1,2,3}$ fixed. Lastly, a fit was made where all parameters were allowed to vary over the entire frequency range. For the result of this fit please see table 1 and table 3.

4. Origin of the Recipe

Here I will explain how I reverified and convinced myself that (Ganya's method)/(the recipe described in section 3) is the proper way to correct for EDIE. XTE J1859 was the primary source used here. However, information about other sources is taken from Ganya's thesis.

Here a description of tests for qpo-like structure in the power spectrum of XTE J1859 will be discussed. Specifically, the problem to be addressed is whether the three Lorentzian structures shown in figure 6 are real or would it be possible to adequately describe the power in that plot using a simple power law void of any structure. The answer to this question hinges on understanding EDIE in the power spectrum.

Table 1. Properties of the Lorentzians

Lorentzian	Centroid Frequency (Hz)	FWHM (Hz)	Amplitude (rms??)
1	$73 + 4 - 4$	$56 + 29 - 26$	$0.035 + 0.031 - 0.015$
2	$132 + 2 - 2$	$14 + 12 - 6$	$0.025 + 0.008 - 0.007$
3	$193 + 16 - 35$	$115 + 10190 - 55$	$0.021 + 0.054 - 0.011$

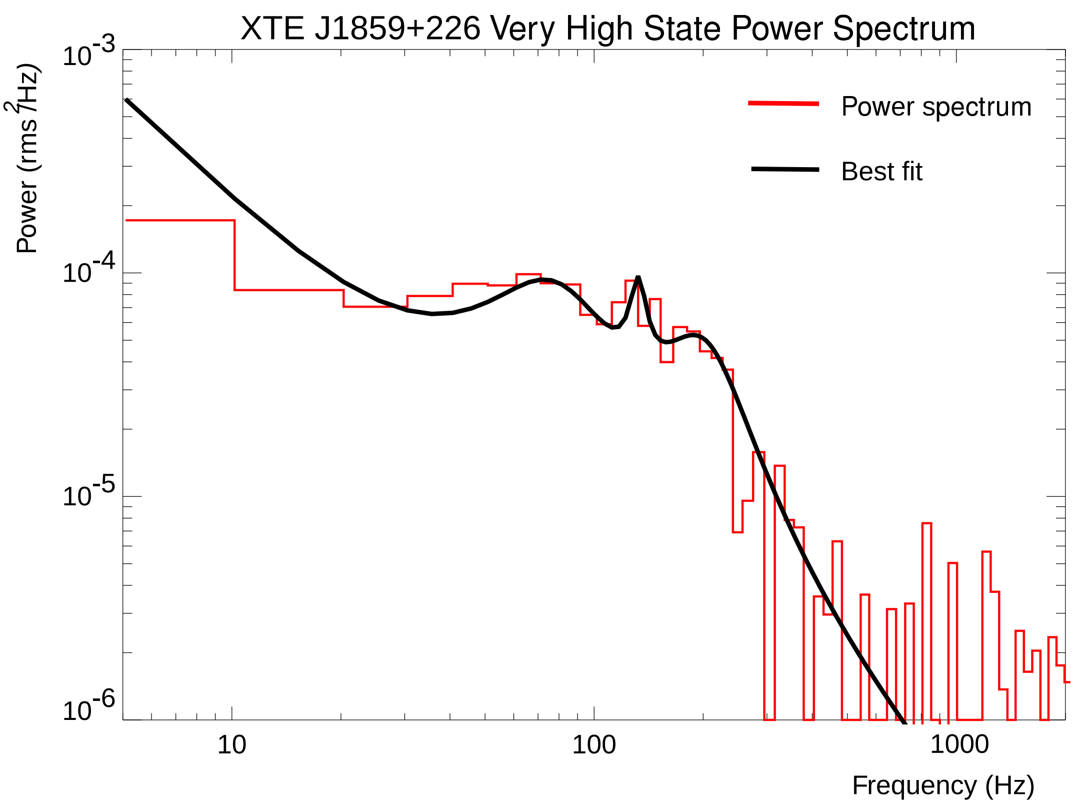

Fig. 6.— Power spectrum of XTE J1859+226 in its VHS. The Spectrum is fit with a power law plus three Lorentzians.

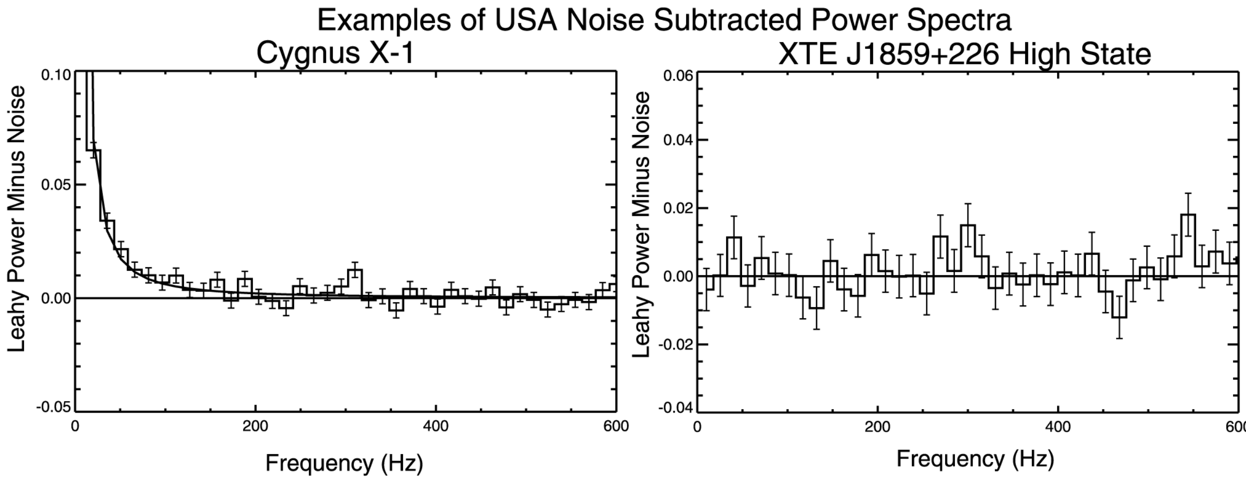

Fig. 7.— Far left: Noise-subtracted power spectrum of the mode 1 Cygnus X-1 data. Energy range: 4.2-19.8 keV. =377 Hz. =96 ms. Total exposure time 23.6 ks. Range used to determine the noise: 320-5000 Hz. Solid line is a power law with index -1.53. Near left; Noise-subtracted averaged power spectrum of XTE J1859+226 during the high state in the energy range 4 to 173 keV. The average observed rate in this energy range is 274 c/s. High-frequency variability is weak or absent in the high state.

Table 2. $5Hz$ QPO Properties

QPO	Centroid Frequency	FWHM (Hz)	RMS (%)	χ^2/dof	dof
Sub-Harmonic	2.6 ± 0.1	0.32 ± 0.27	2.2 ± 0.8	0.56	16
Fundamental	4.89 ± 0.04	0.46 ± 0.1	4.9 ± 0.4	1.17	33
First-Harmonic	9.84 ± 0.08	0.4 ± 0.2	2.3 ± 0.5	0.93	69

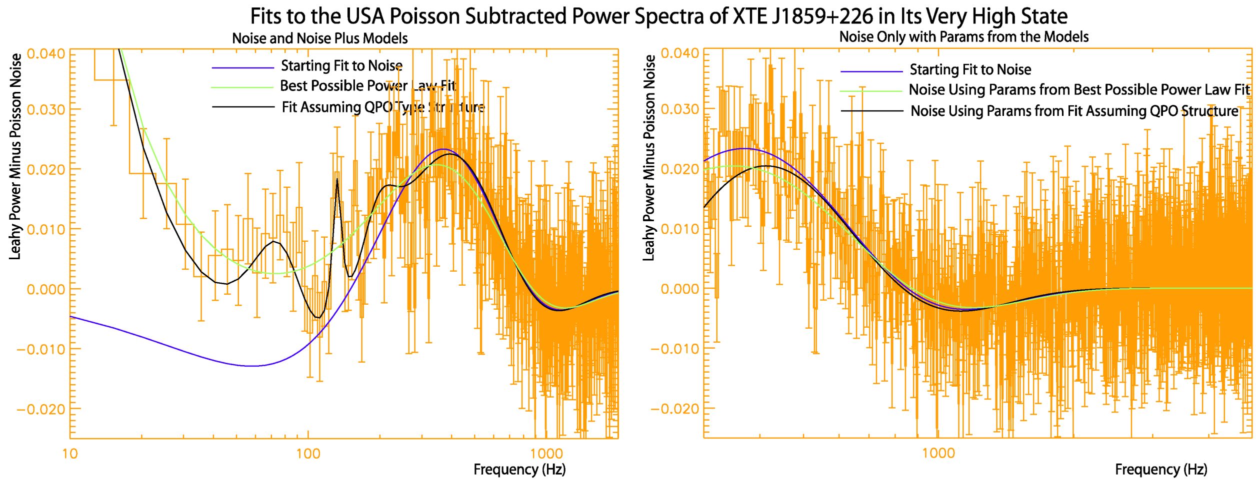

Fig. 8.— The two plots show some of the work done to address a subtle question about the USA instrumental effect correction. This question is where is it acceptable to fit an assumed power spectral structure. The answer is that once a the correct structure is determined it should be possible to vary all parameters of the entire frequency range. One should find that correct model using the process described in the following paragraph. The consequence of fitting an incorrect model to the entire frequency range is that the equation **??** parameters are distorted and they will no longer accurately describe the USA instrumental effect. Above one sees that the pure power law,green, pulls the fit away from the equation **??** line, purple, more severely than the Lorentzian.

Table 3. Result of Fit to Equation 7

Parameter	C_0	C_1	C_2	C_3	B	Normalization	Index
Fit Value and One Sigma Error	$-4.32e-9^{+1.04e-9}_{-2.02e-9}$	$261.08^{+186}_{-74.4}$	$949.48^{+53.1}_{-53.1}$	$1.140^{+0.1}_{-0.08}$	1.0 Fixed	$3.526^{+0.927}_{-0.600}$	$-1.579^{+0.1}_{-0.2}$

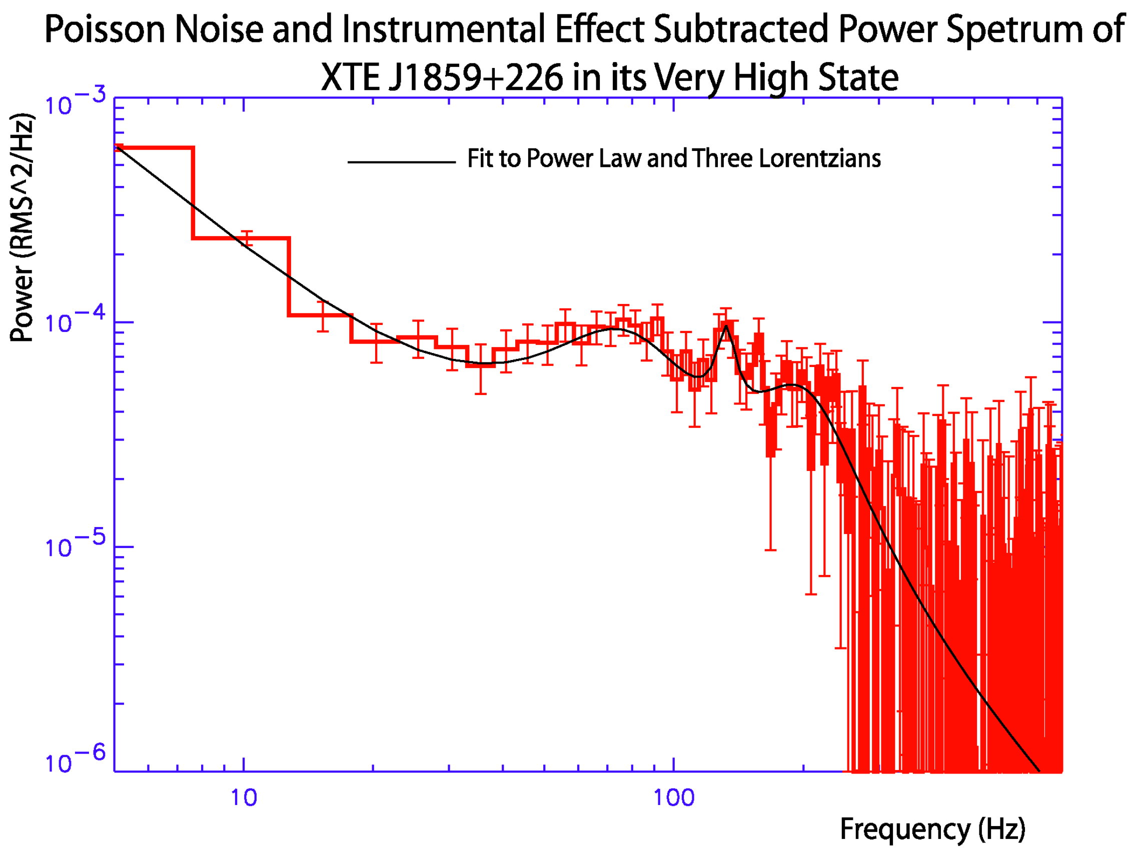

Fig. 9.— This is same as figure 6 except that the data is not re- binned. All fitting was done using the original bins used in the fft's. This plot represents an average of 2^{11} bins $96e-6$ seconds in width over 13.4 kiloseconds of data. Data was only used from the energy range 4 to $17 \pm 3keV$. The average observed count rate in this energy range is 465 c/s.

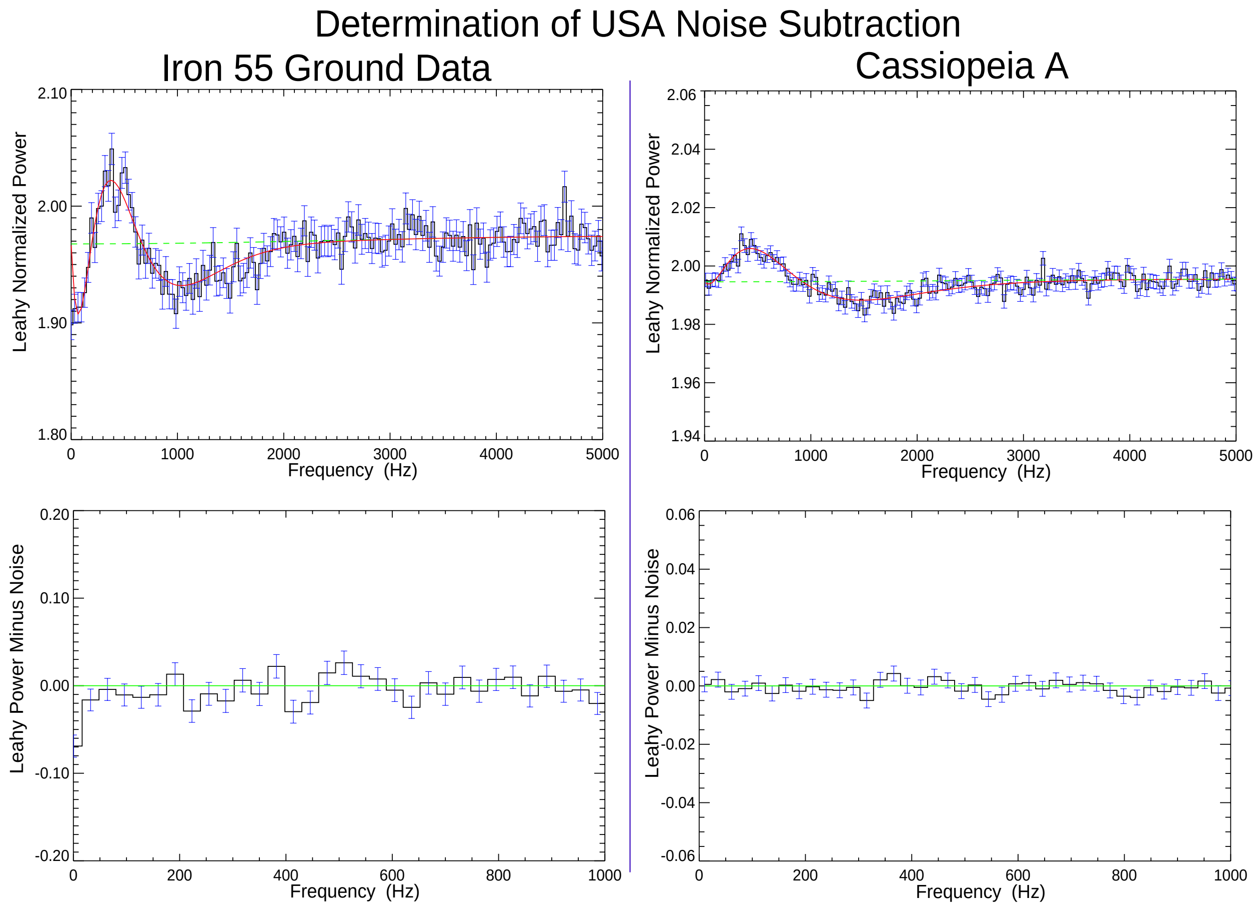

Fig. 10.— Top Left: Power spectrum of the USA mode 2, detector 0, channel 6 ground data fitted with Rate= $523Hz$, Binsize= $96\mu s$. Total exposure time $0.7ks$. Fitting rage: $0-5000Hz$. Solid line is the fit, The dashed green line is the simple Poisson noise floor with dead time effects. This is equation 8. Bottom Left: Residuals of the fit to the ground data power spectrum after subtracting the noise floor. Fitting range: $300-5000Hz$. The residuals are consistend with zero in $\approx 30-5000Hz$ region. Top Right: Power spectrum of the mode 1, channel 2 Cassiopeia A data fitted with equation 1. Rate=106 Hz. Binsize=96 ms. Total exposure time 22.7 ks. Fitting range: 300-5000 Hz. Dashed green line is is equation 2. Bottom Right: Residuals of the fit to the Cas A power spectrum with equation 6. Fitting range: 300-5000 Hz. The residuals are consistent with zero in 0-5000 Hz region.

By making a power spectrum for each of USA's 16 energy channels for modes one and two data from the Fe^{55} source, Caseopiea A, Cygnus X-1, the Crab Pulsar and XTE J1859 we were able to learn the qualitative behaviour of EDIE. The lesson of Ganya's, Gary Godfrey's and my own work on the instrumental effect is that it has a particular and well defined shape which is generally predicted by a source's count rate and energy spectrum. This information lead Gary to believe that EDIE is caused by pile up in the electronics. This theory seems to still be correct.

A sketch of the procedure for characterizing the energy-dependent effect in USA power spectra is as follows: Using ground calibration data the general functional form and qualitative energy-dependence of the effect was determined. Making a fit to mode 2 channel 6 ground data, figure 10, in the frequency range of $0 - 5000Hz$ the functional form of the instrumental effect was found to be: **??**

$$P^{noise}(f) = P^{instrumental}(f) + bP^{other}(f) \tag{6}$$

where

$$P^{instrumental}(f) = c_0 e^{-0.0048f'} f'(f' - c_1)(f' - c_2), \tag{7}$$

$$P^{other}(f) = P_1 + P_2 \cos(2\pi f t_b) \tag{8}$$

and

$$f' = c_3 f \tag{9}$$

Note P^{other} is the Poisson noise floor as modeled in Ganya's thesis and the parameters are defined there. P^{other} is the simple Poisson noise floor accounting for deadtime and a few minor idiosyncrasies of the USA detector electronics. The value of -0.0048 in the exponential was found through fitting and is thought to be a time constant of the electronics which are causing EDIE. The final outcome of fitting this equation to the ground data was an excellent fit in the region from a $30 - 5000Hz$.[3] The fit only varies from the power spectrum in the region below $30Hz$. This is completely adequate when looking for signals at higher frequencies. However, it should be noted that for Cas A Cyg X-1 and XTE J1859(high state) the expected results are obtained, even from $0 - 30Hz$ when using equation 6 to subtract the noise and instrumental effects. Please see figures 10 and 7.

Once an equation describing EDIE that was able to fit most or all of USA power spectra was found there was a question of how to use it. There is, according to Ganya's thesis, precedent for fitting the tail of a power spectrum and then using that model as a noise subtraction for the unfitted part of the power spectrum. However, I didn't know that before I did all of the following work. So, I did quite a bit of work which I believe verifies the recipe given in section 3.

The first step in the application of this phenomenological model of EDIE was obvious in all cases. First, a given power spectrum is fit in the range of $300 - 5000Hz$, or where it is believed there is no structure, to equation 6[4]. Doing so should determine the parameters of P^{noise} in the range of $0 - 5000Hz$.

The next step wasn't as obvious. Yet it is extremely important. In XTE J1859, for example, depending on the recipe used for subtracting off EDIE, one can rule out a pure power law between $1 - 300Hz$ with $68\% - 93\%$ confidence.

Given this information it is possible to determine how well a power law plus three Lorentzians fits the XTE J1859 data shown in figure 6 of Ganya's paper. Specifically, a

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[3]Please see **??**.

[4]The range of the fit depends on the ranges of interest. It appears that it is possible to get a very accurate model of the noise floor by fitting at higher frequencies, for example $500 - 5000Hz$

comparison needs to be made between a pure power law fit and a power law with three Lorentzians. Here it will be shown that a pure power law may be excluded at the 68% confidence level in the worst case scenario and at the 93% confidence level in the best case. To make this comparison a variety of possible fitting scenarios will be explored.

Ganya's original unbinned power spectrum was fit using both a pure power law and and a power law plus three Lorentzians. The results of these fits are shown in figures 11-20.

The general recipe for making these fits will now be given. First equation 8 was subtracted from the power spectrum. Then a fit was made of the resulting spectrum from $300 - 5000Hz$ using equation 7. Next the power spectrum was fit with either a power law or a power law plus three Lorenztians added to equation 7 in the range $1 - 300Hz$ or $1 - 5000Hz$. These fits are described in detail in the caption of each figure.

Given even the incomplete knowledge of the instrumental effect demonstrated in Ganya's thesis it is possible to exclude,by eye, the fitting scenarios, used to characterize the combined instrumental effect and source variability, used in figures 14, 17 and 18. These plots are excluded simply because it is clear that the parameters which were allowed to vary in the low frequency range should not have been. Their variation incorrectly pulled the fit of the instrumental effect off.

Further, making use of the limited understanding of the instrumental effect it is possible to eliminate any scenario where the parameters of equation 7, which effect generic shape of the instrumental effect, are allowed to vary in the low frequency range. This excludes figures 16 and 20 even though both of these fits give good χ^2s, 1.12 and 1.13, respectively. The problem is these two fits were done over the entire frequency range. Thus, the good χ^2 are more a nice testament to the knowledge of the instrumental effect than they are a deciding factor between a pure power law and structure at low frequencies.

Comparing figures 11, 13, and 15 as well as figures 12, 17 and 19 argues the case that a correct instrumental effect subtraction is not obtained if one varies the amplitude of equation 7 at only low frequencies. This is because, one can see from the figures, if the amplitude c_0 is allowed to vary in only the low frequency fit, then it pulls the fit off of the known shape of EDIE. This allows the exculsion of the fitting scenarios in figures 13 and 17.

There are now only two fitting recipes remaining. These two scenarios are the ones shown in figures 11 & 12 and figures 15 & 19. By the same reasoning used to exclude figures 16 and 20, figures 15 and 19 are also excuded based on their χ^2 values.

However, with consideration to trying to prove structure above a power law in XTE J1859, figures 15 and 19 may be thought of as a worst case scenario. They have artificially low χ^2 values because of the very good fit of EDIE at high frequency and very limited knowledge of the structure present at low frequency(i.e. the goodness of the fit at high frequencies is outweighing the badness of the fit a low frequencies). In this worst case scenario, for the fit in figure 19 $\chi^2 = 1.117$ and for figure 15 $\chi^2 = 1.153$. Using an f-test:

$$\frac{1.153}{1.117} = 1.032 > 1.030 \tag{10}$$

for 1021 degrees of freedom in the numerator and 1012 degrees of freedom in the denominator. The value 1.030 is the value at which a 68% confidence level exclusion of a pure power law may be made.

Now looking at what I consider to be the correct way to make the fits, where all equation 7 parameters are frozen, including c_0, after fitting to the high frequency region, it is possible to exclude the pure power law at a 93% confidence level. This is done using figures 11 and 12. The χ^2 values for the fits shown in figures 11 and 12 are 1.934 and 1.281, respectively. Here fits were first made at high frequencies, all noise parameters were then frozen and a fit was made to any structure at low frequencies. This is just as in section 3.

Finally an f-test is used to compare the fits in figures 11 and 12:

$$\frac{1.934}{1.281} = 1.510 > 1.509 \tag{11}$$

for 56 degrees of freedom in the numerator and 47 degrees of freedom in the denominator. The value 1.509 is the value at which a 93% confidence level exclusion of a pure power law may be made.

This is exactly the same method Ganya used to obtain the corrected power spectra for Cas A Cyg X-1 Crab in the $0 - 300 Hz$ frequency range. One significant addition made through testing and verifying this process is a means to estimate the systematic uncertainties due to the lack of knowledge of EDIE. So far the best way to obtain proper error bars on the fit parameters is to make one final fit varying all parameters, noise and structure.

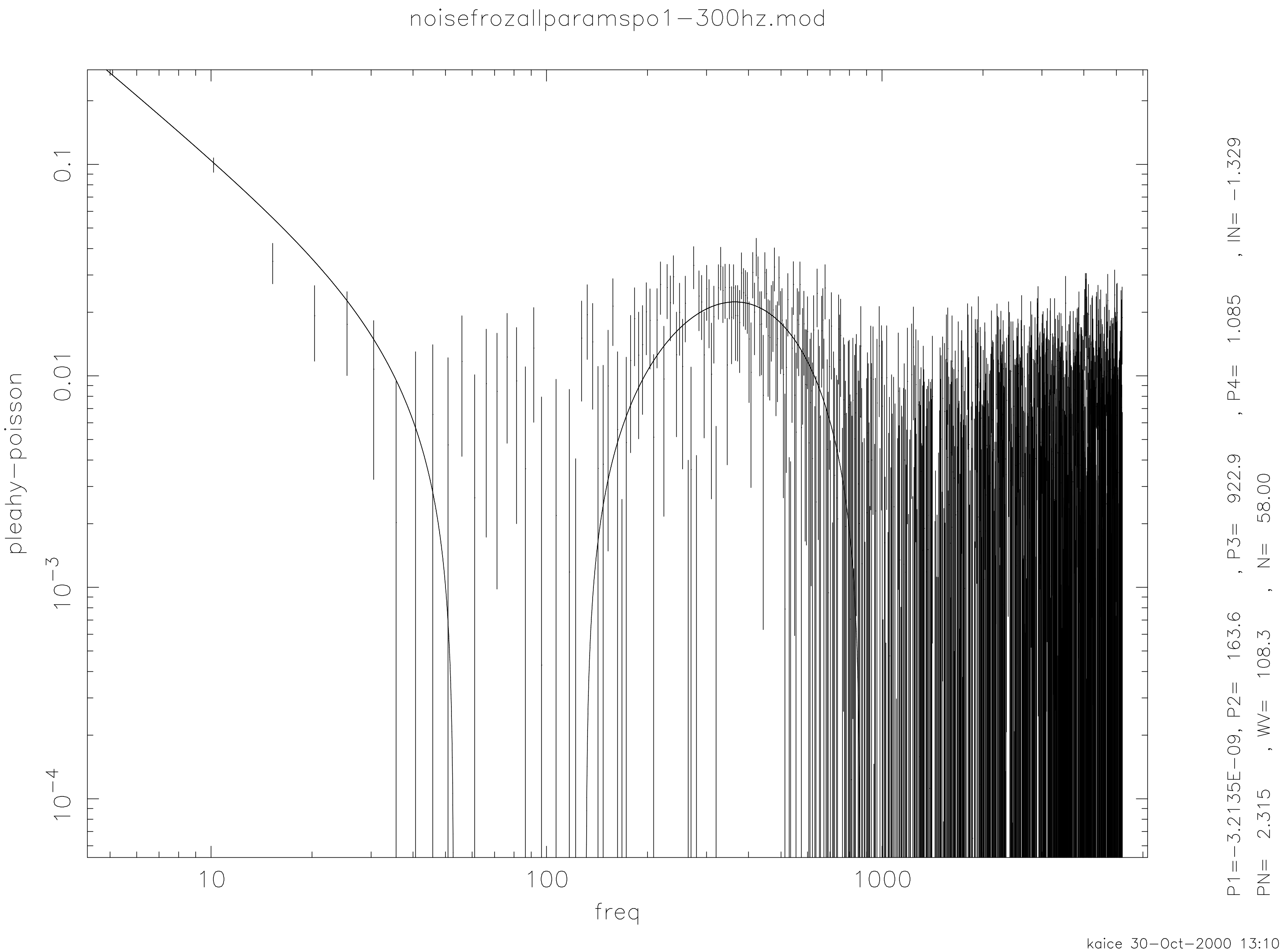

Fig. 11.— equation 7 fit from $300 - 5000 Hz$. All equation 7 Params Frozen. Power Law Plus equation 7 fit from $1 - 300 Hz$

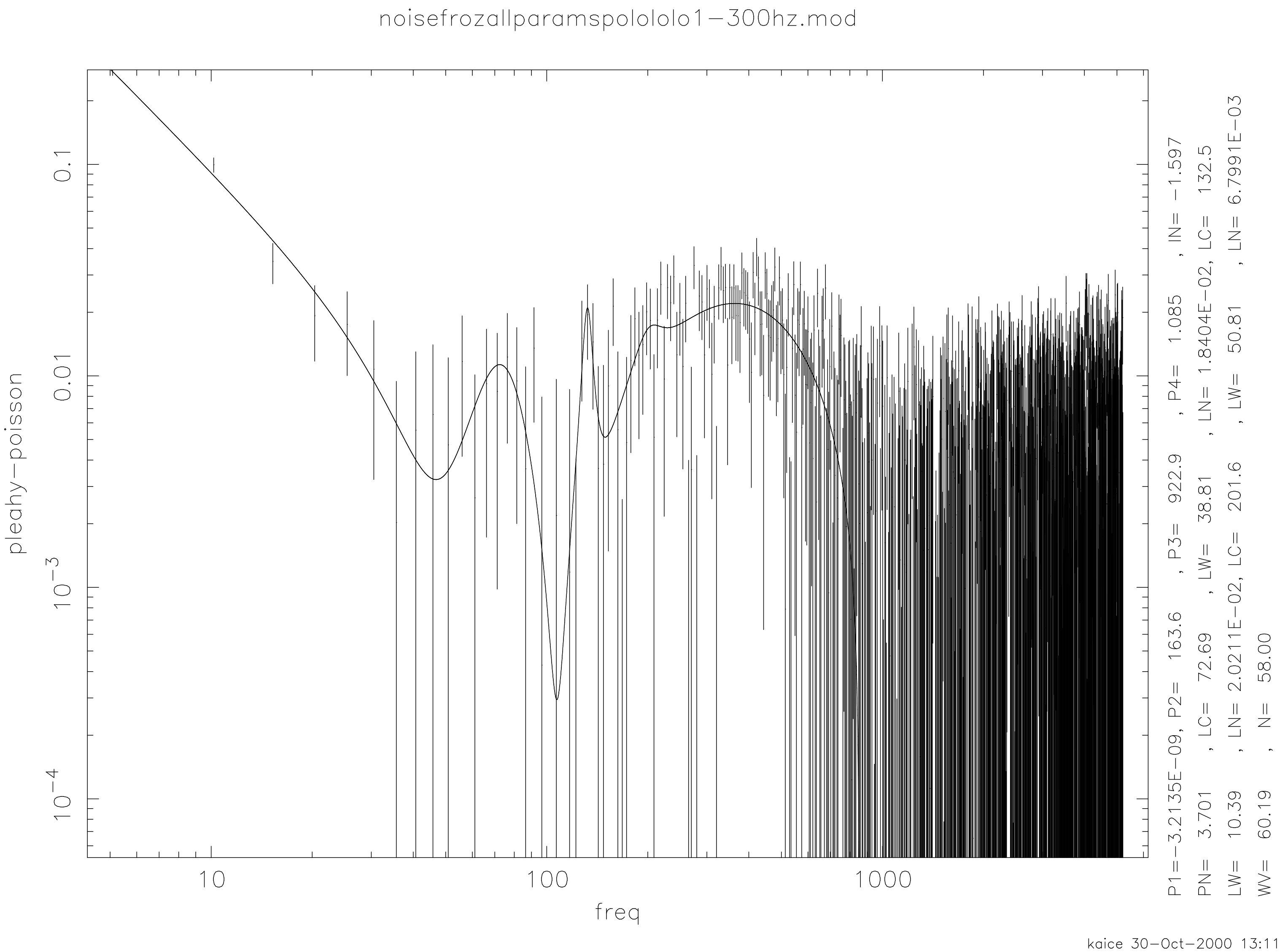

Fig. 12.— equation 7 fit from $300 - 5000Hz$. Then all equation 7 Params Frozen. Power Law and three Lorentzians Plus equation 7 fit from $1 - 300Hz$

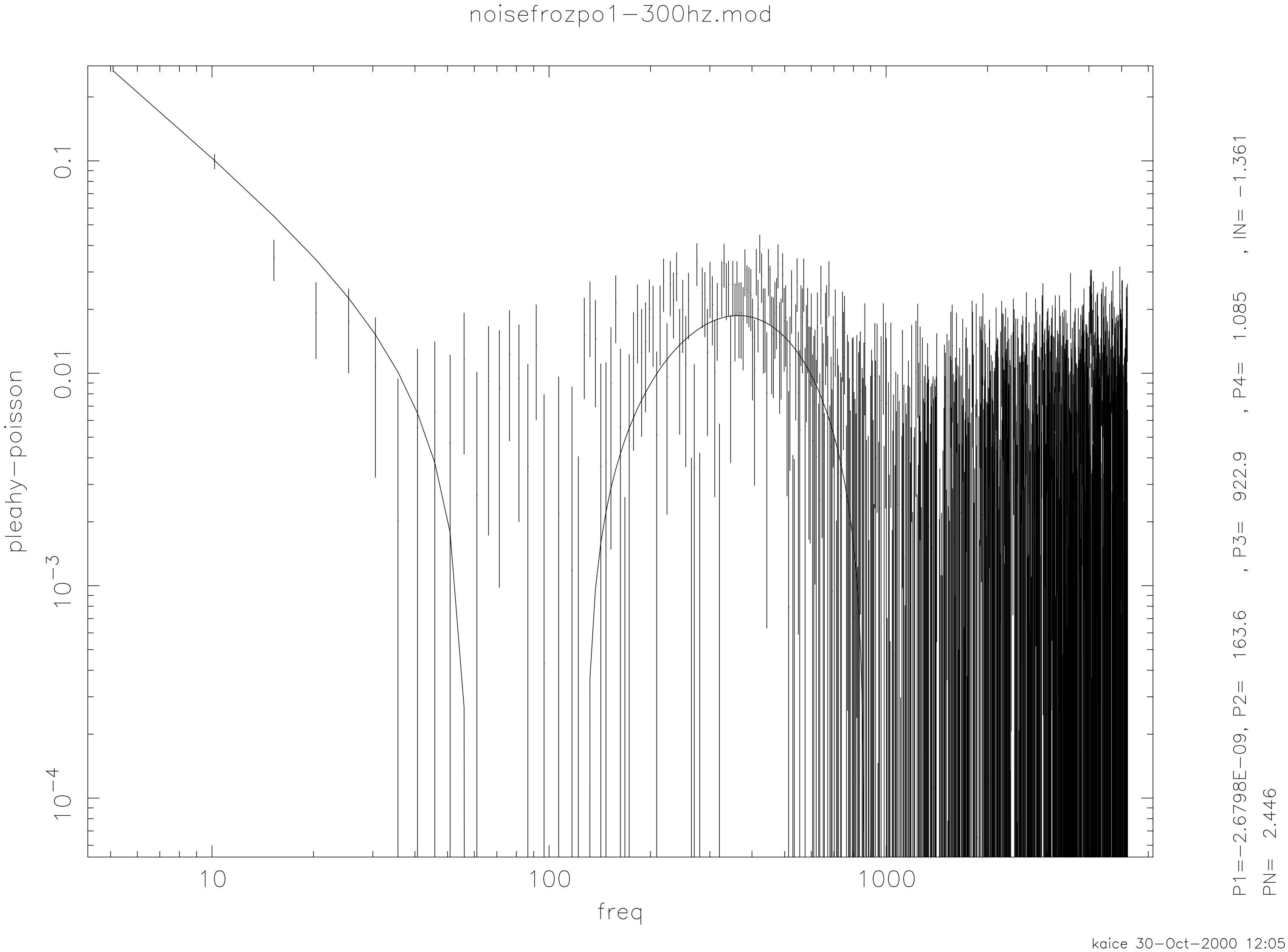

Fig. 13.— equation 7 fit from $300 - 5000 Hz$. Then all equation 7 Params except c_0 Frozen. Power Law Plus equation 7 fit from $1 - 300 Hz$

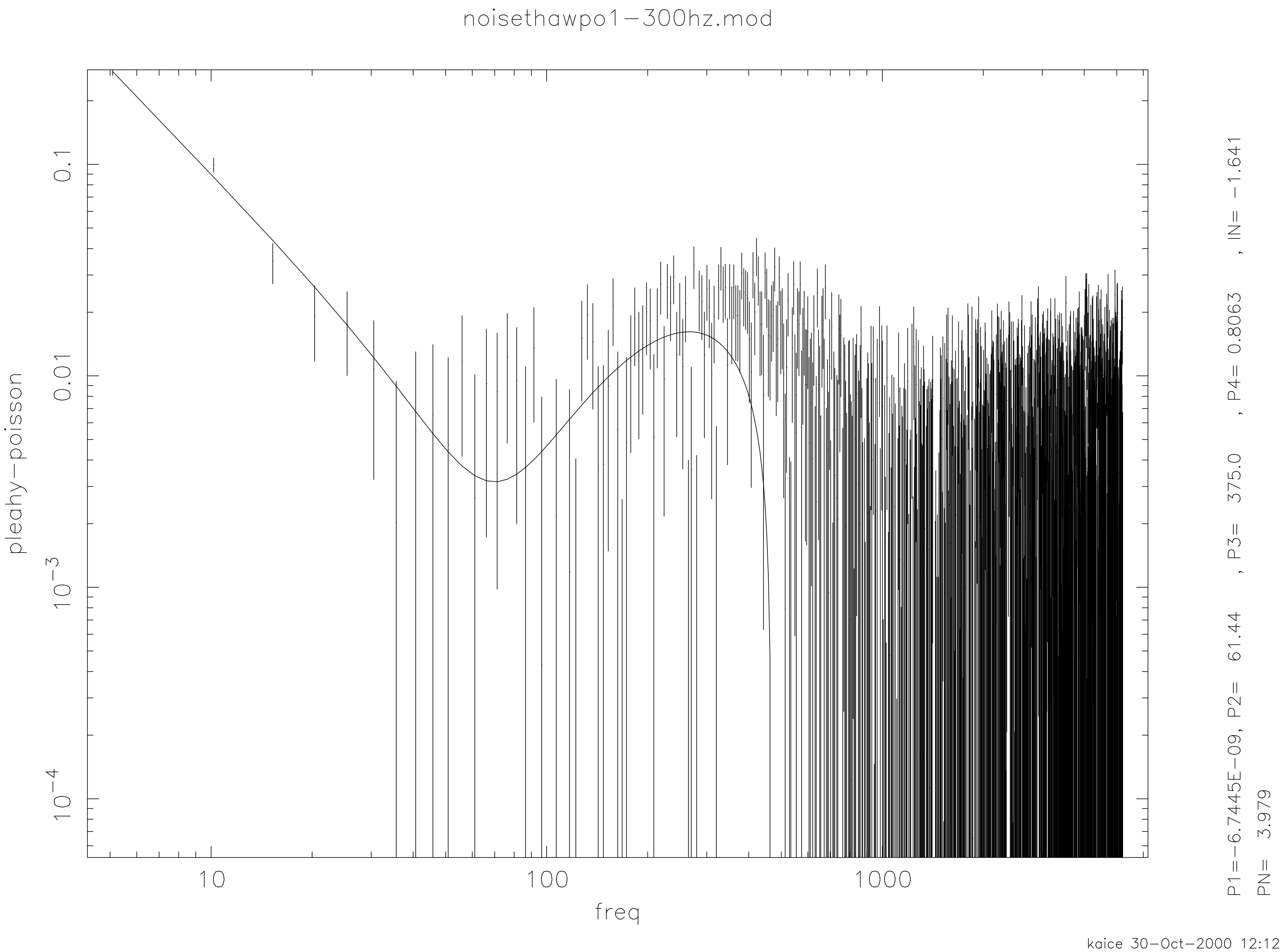

Fig. 14.— equation 7 fit from $300-5000Hz$. No equation 7 params frozen. Power Law Plus equation 7 fit from $1-300Hz$

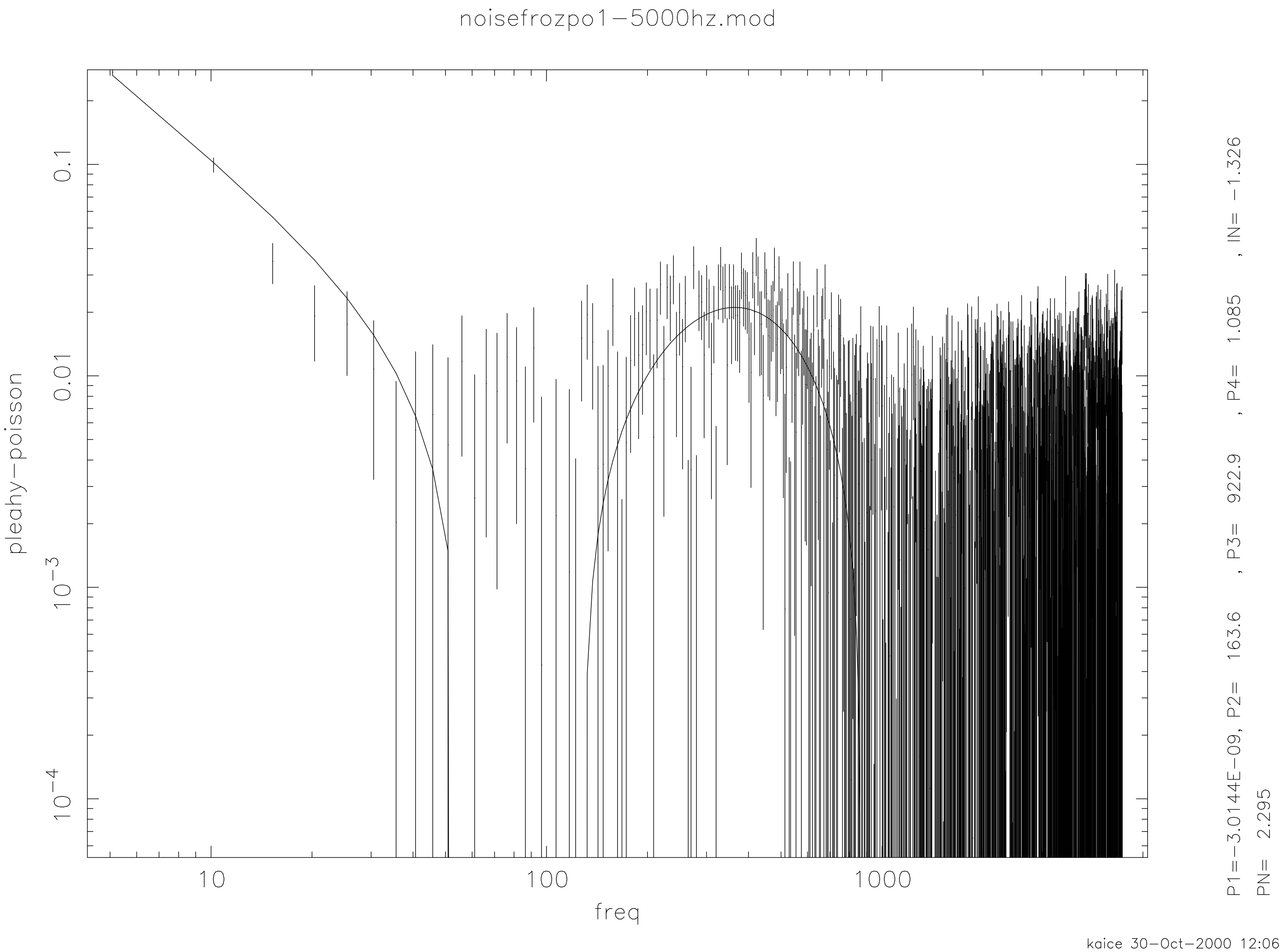

Fig. 15.— equation 7 fit from $300 - 5000 Hz$. Then all equation 7 except c_0 Params Frozen. Power Law Plus equation 7 fit from $1 - 5000 Hz$

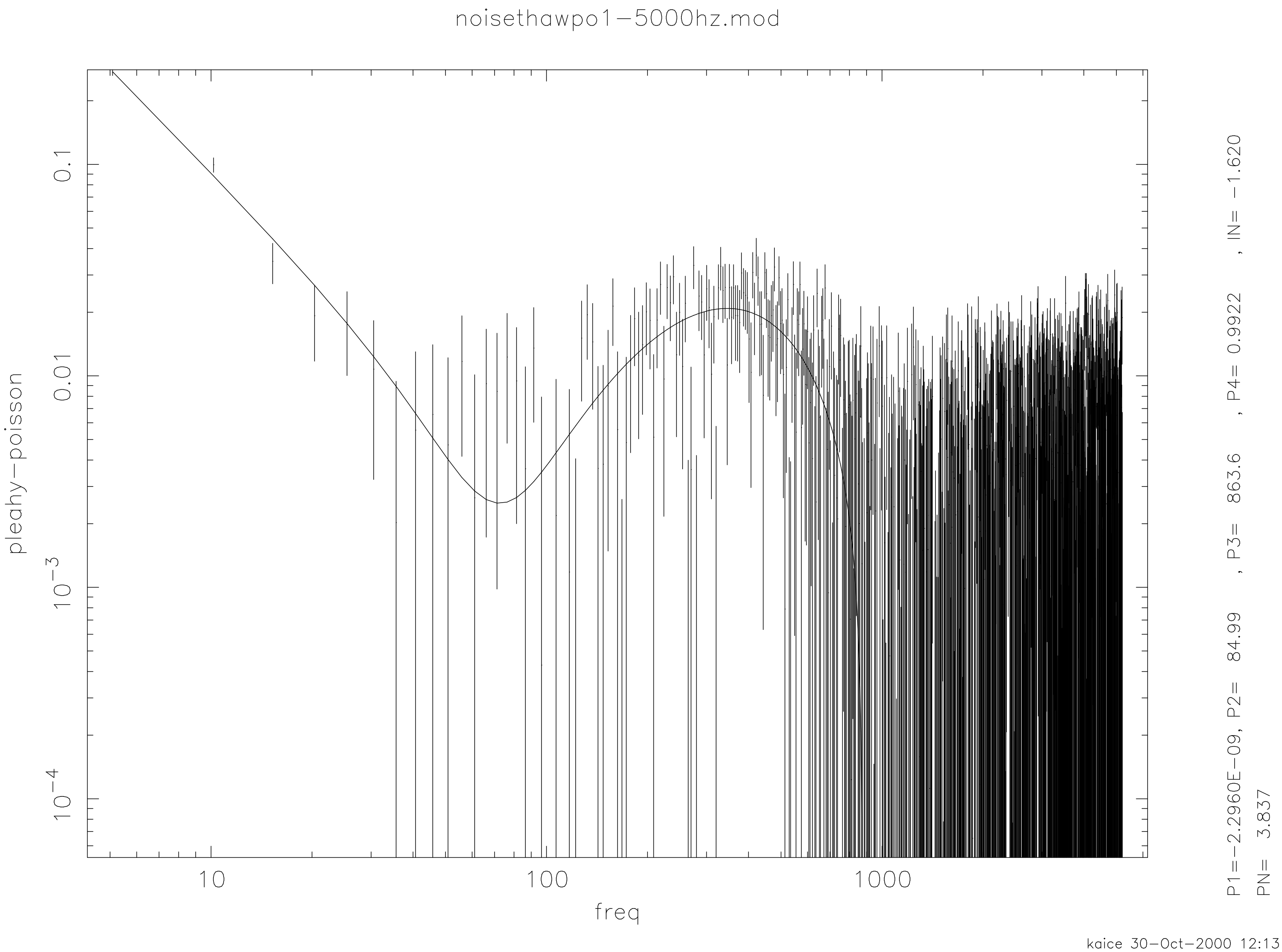

Fig. 16.— equation 7 fit from $300 - 5000Hz$. No equation 7 params frozen. Power Law Plus equation 7 fit from $1 - 5000Hz$

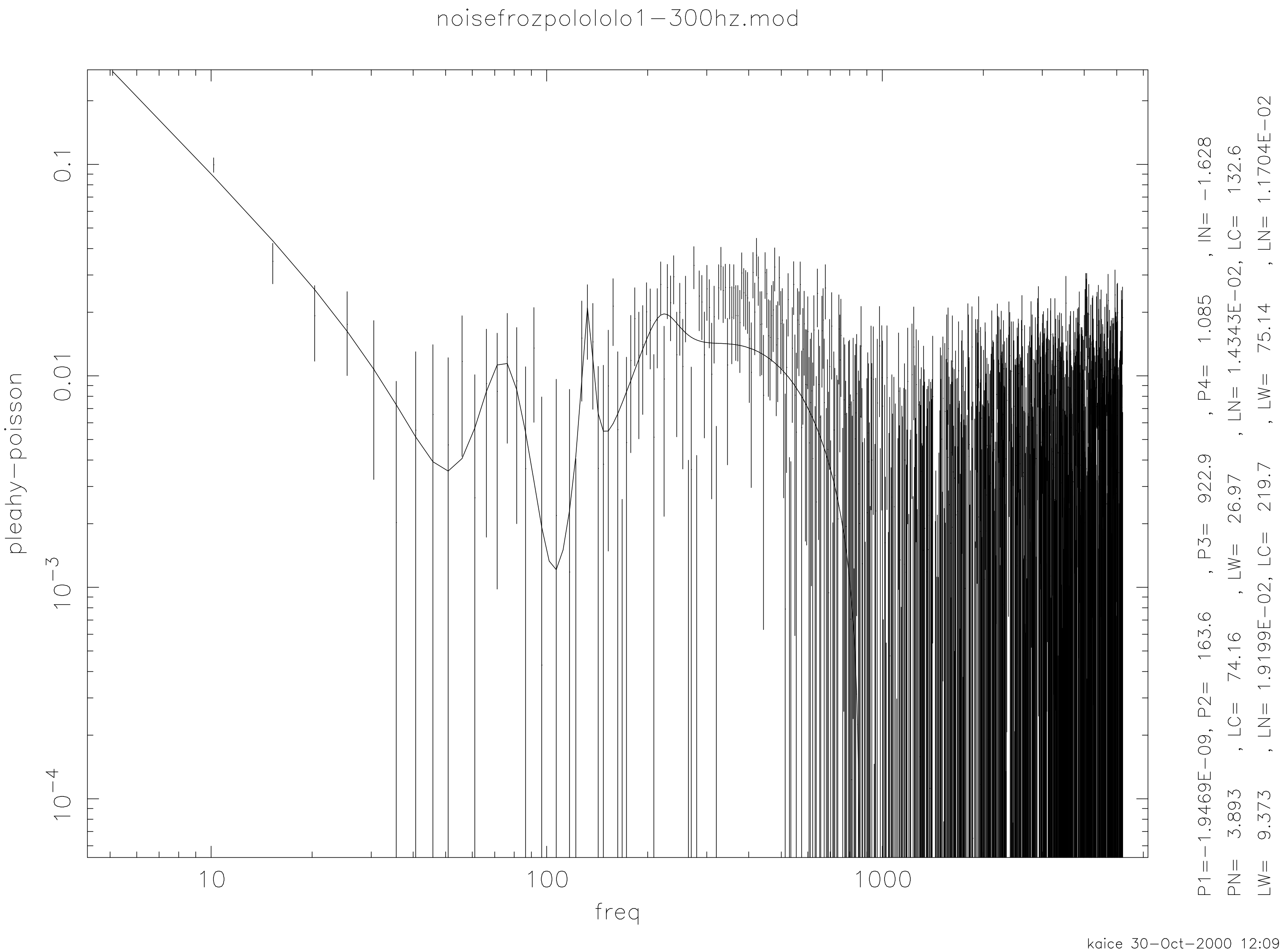

Fig. 17.— equation 7 fit from $300 - 5000Hz$. Then all equation 7 except c_0 Params Frozen. Power Law and three Lorentzians Plus equation 7 fit from $1 - 300Hz$

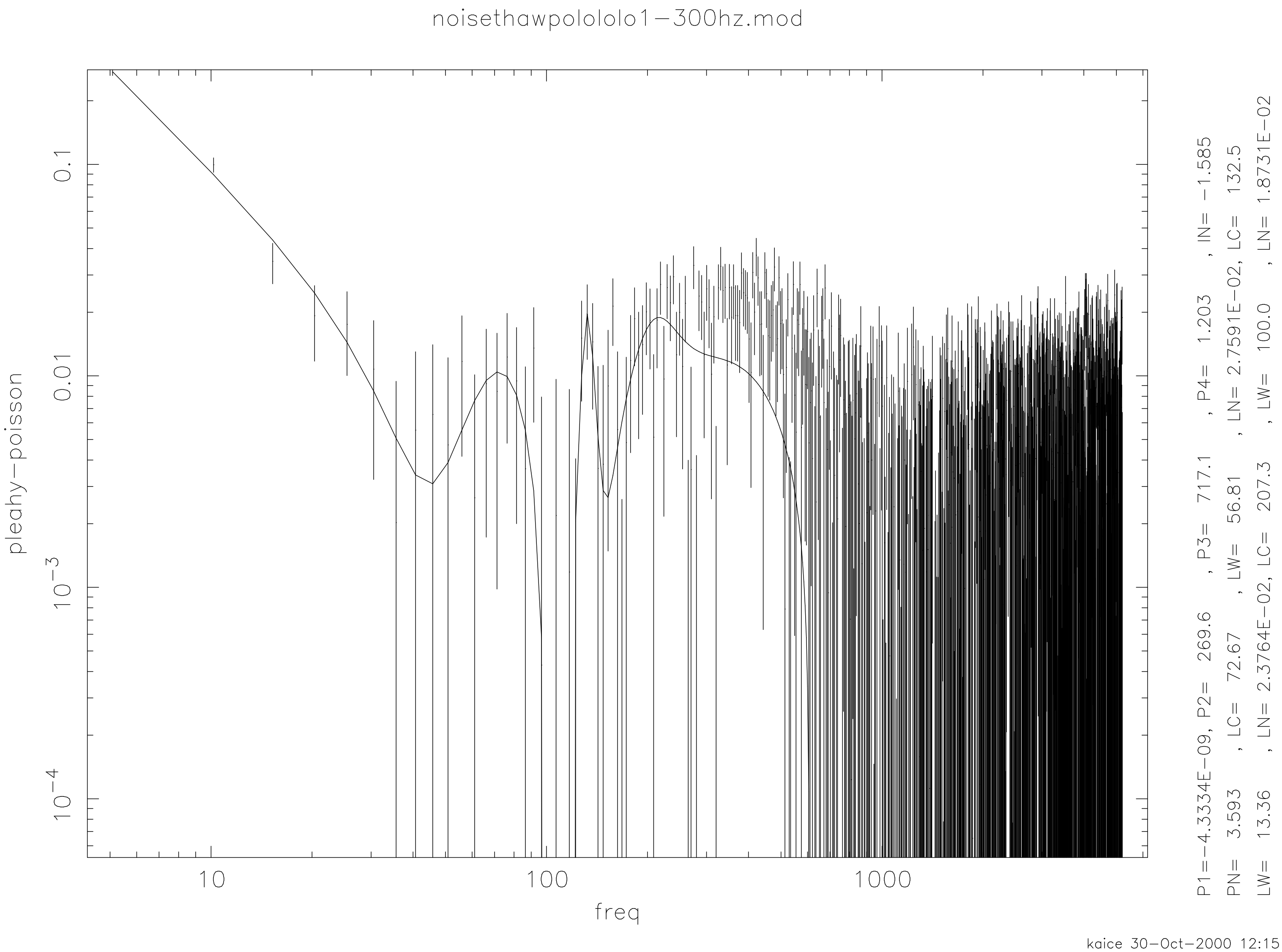

Fig. 18.— equation 7 fit from $300 - 5000Hz$. No equation 7 params frozen. Power Law and three Lorentzians Plus equation 7 fit from $1 - 300Hz$

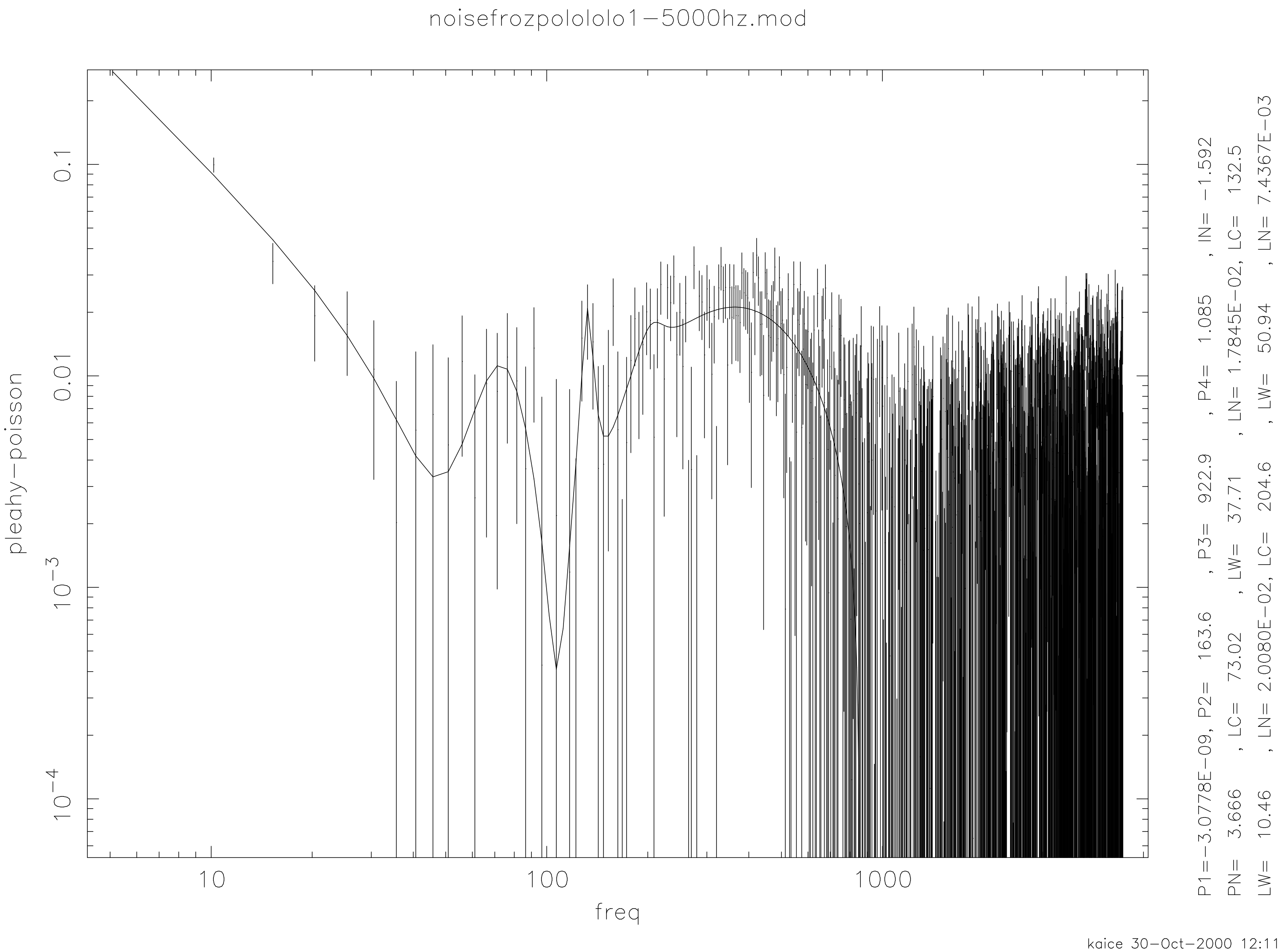

Fig. 19.— equation 7 fit from $300 - 5000 Hz$. Then all equation 7 except c_0 Params Frozen. Power Law and three Lorentzians Plus equation 7 fit from $1 - 5000 Hz$

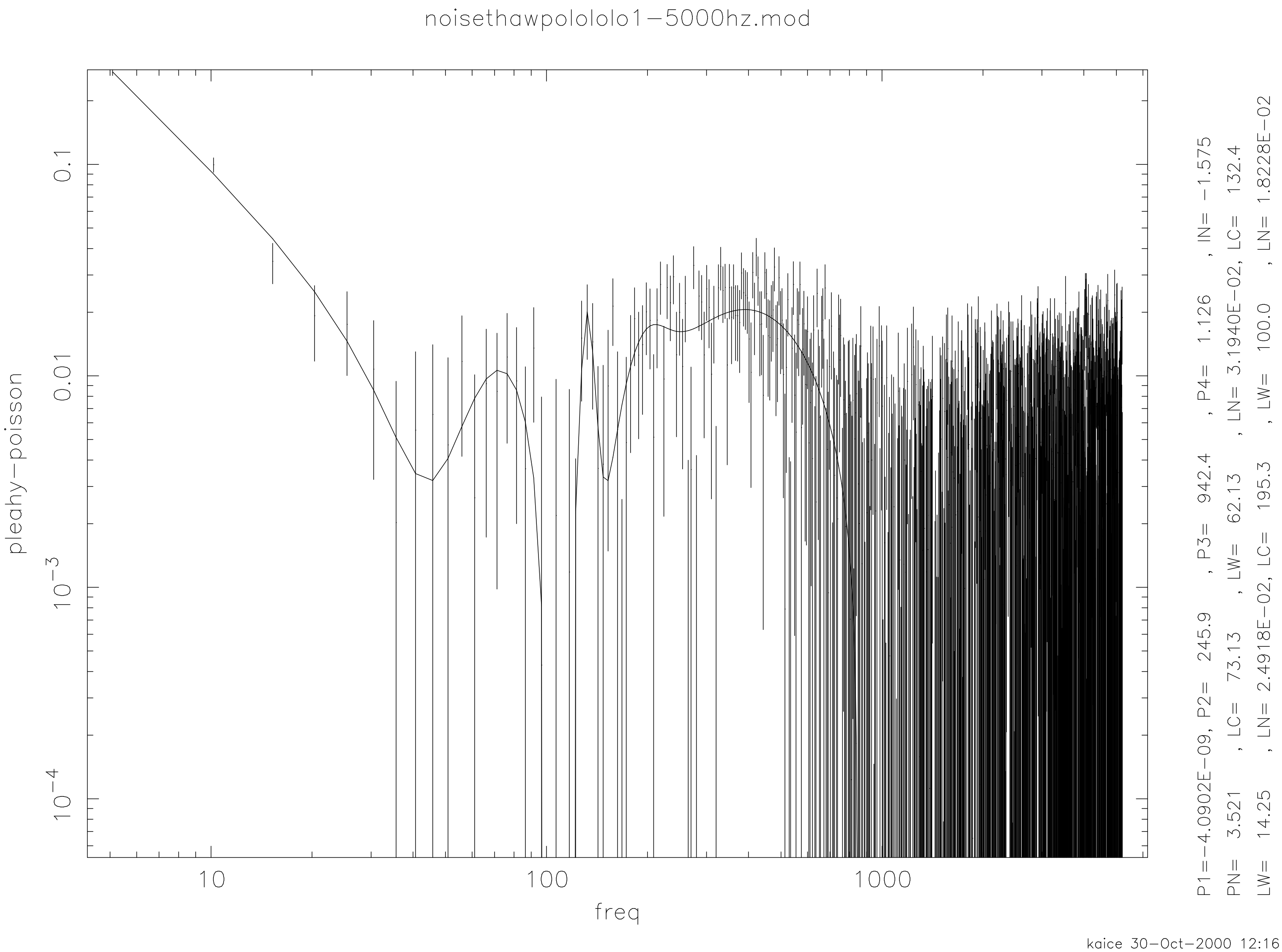

Fig. 20.— equation 7 fit from $300 - 5000Hz$. No equation 7 params frozen. Power Law and three Lorentzians Plus equation 7 fit from $1 - 5000Hz$